very fine minds incapable of following mathematicial demonstrations. Need we add that mathematicians themselves are not infallible?

"The answer seems to me evident. Imagine a long series of syllogisms, and that the conclusions of each serve as premises for the following. Between the moment in which we first meet a proposition as a conclusion of one syllogism, and that in which we re-encounter it as the premise of another, occasionally some time will elapse... so it may happen that we have forgotten its meaning. We replace it by a slightly different proposition or attribute to it a slightly different meaning, and thus it is that we are exposed to error".

This is the danger of the logical step-by-step approach: we lose sight of the total picture. The point is made more succinctly in a passage from the writer Stefan Themerson: "Only an elephant or a whale gives birth to a creature which weighs over 100 kg. The president's weight is 120 kg. Therefore the president's mother was either an elephant or a whale".

Poincaré also describes his experiences with a particular research problem. "For fifteen days I strove to prove that there could not be any functions like those I have since called Fuchsian functions. I was then very ignorant. Every day I seated myself at my work table, stayed an hour or two, tried a great number of combinations, and reached no results.

"One evening, contrary to my custom, I drank black coffee and could not sleep. Ideas rose in crowds: I felt them collide until pairs interlocked, so to speak, making a stable combination. By the next morning I had established the existence of a class of Fuchsian functions: I had only to write out the results, which took a few hours".

Poincaré concludes that mathematical creation involves four stages, which we might call preparation, incubation, illumination and verification. Preparation is a period of conscious work, often apparently fruitless. Incubation requires a period free of conscious thought, during which time the subconscious 'plays' with the problem. This leads to illumination: an idea strikes 'out of the blue' accompanied by a feeling of certainty. Finally, there is a period of conscious verification that the idea is in fact correct. (The process may, of course, get stuck or go wrong at any stage.)

Other mathematicians have had similar experiences. J.E. Littlewood relates that he had struggled for two months to prove a result that he was pretty sure was true. Half way up a Swiss mountain, a very odd device emerged — so odd that though it worked, he could not grasp the proof as a whole. He had a sense that his subconscious was saying "Are you *never* going to do it, confound you? Try this!"

Different mathematicians have different talents. William Thurston, at Princeton, has been described as a "geometrical wizard". He waves his hands and wonderful things materialize from nowhere. Others, such as John Horton Conway at Cambridge, are especially good at unravelling combinatorial complexities. As a result, mathematicians spend a lot of time talking to each other, in the hope that one may see clearly through what to the other is impenetrable fog. I once witnessed a discussion between two colleagues about how to turn a sphere inside out. It lasted about an hour, was carried out over the telephone, and one of them was blind. They must have developed their own very personal set of images, but it was marvellous discipline. If you can explain topology over the telephone to a blind man you must understand it rather well.

The point that this series of anecdotes drives home is that the way mathematicians think about their problems is very different from the way that they present their formal results. It is the latter that has given mathematics a reputation for being austere and dry, but it represents the surface, rather than the essence, of the subject. Although there may be advantages in appearing arcane and impenetrable, I rather think that the disadvantages carry more weight, the main one being that nobody outside the magic circle has the faintest idea what you are doing, or indeed that you are doing anything.

I am not suggesting that mathematicians start writing research papers including a potted history of all the blind alleys that they blundered into along the way. Journal space is finite. But it would be wonderful if some of the great mathematical minds of today would share with the rest of the world a little more of this inside story. If mathematicians can come to terms with the fact that their subject is the work of human beings, then this planet of human beings may become more favourably disposed towards mathematics. □

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Low-temperature physics

Vortex production in helium

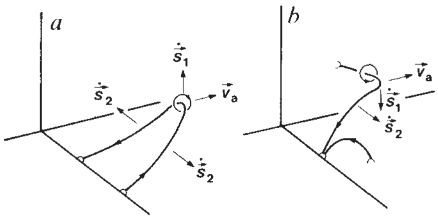
Peter McClintock

THE celebrated frictionless flow properties acquired by liquid helium at very low temperatures have long been known to vanish abruptly if the velocity of the liquid exceeds a certain critical value. The physical mechanism underlying this breakdown of the superfluidity has, however, remained an enigma, and it is pleasing, therefore, that a possible solution to this important and long-standing problem has now been proposed by K.W. Schwarz of IBM (*Physical Review Letters* **57**, 1448; 1986). The timing of his contribution is especially opportune in view of the current illuminating experimental work of E. Varoquaux (Université de Paris-Sud, Orsay) M.W. Meisel and O. Avenel (Centre d'Etudes Nucléaires de Saclay), some results of which are also published in a recent issue of *Physical Review Letters* (**57**, 2291; 1986). The latter authors are studying the onset of dissipation for an oscillating flow of superfluid helium through a very tiny orifice, using a technique which provides unprecedented sensitivity.

The basic experimental observation, which has been confirmed by many researchers, is that liquid helium at very low temperatures can pass at moderate speeds through tubes or orifices without any energy dissipation. The viscosity of the liquid is essentially zero, and so the flow can be maintained with no need of the pressure gradient that is required to keep a conventional liquid moving. But when the flow exceeds a characteristic critical velocity, which usually seems to increase as the dimensions of the flow channel are reduced, there is a sudden onset of dissipation. It is generally accepted that this dissipation corresponds to the production within the liquid of quantized vortex lines, the quantum version of ordinary classical turbulence, by some unknown means. Any given element of vortex line can be envisaged as a narrow core region of atomic dimensions around which the superfluid flows at a tangential velocity that is inversely proportional to the distance from the centre of the core. Vortices within the liquid do not have free ends (which would be unstable) but, depending on circumstances, they can exist as isolated lengths stretched between the walls of the vessel; or as closed rings or other shapes within the body of the fluid; or as a dense tangle in which each line is eventually joined back on itself. Where pieces of vortex core approach, the flow field around each of them will influence the others and cause them to move.

In a remarkable experiment, Avenel and Varoquaux demonstrated convincingly (*Physical Review Letters* **55**, 2704; 1985) that the dissipative flow of superfluid helium through a very small rectangular orifice takes place in a sequence of discrete events. It is natural to suppose that these correspond to the appearance or movement of individual closed loops of vortex within the orifice. If so, one is bound to ask the fundamental question of whether they grow from elements of vortex already trapped in the orifice (even in the absence of flow) or whether, on the contrary, each is created *ab initio* in a spontaneous transition within a vortex-free volume of the liquid — a very different kind of process.

Schwarz in his new work points out that a spontaneous transition is inherently improbable given the size of the orifice which, although microscopic (5 μm × 0.3 μm), is nonetheless enormous in terms of atomic dimensions. Such a transition would clearly require simultaneous, coherent, changes in the motions of a huge number of atoms. It is more probable, he suggests, that the dissipation arises from the motion or growth of vortices already present in the liquid, as it is well established that even quiescent, non-flowing, superfluid helium contains a few such vortex lines. Both the expansion of vortex lines and their motion across the flow



Behaviour of pinned vortex lines in flowing superfluid helium-4, as deduced on the basis of a digital simulation (from Schwarz, K.W. *Physical Review Letters* **57**, 1448; 1986). *a*, Dissipation; *b*, vortex regeneration. In each case, v_a represents the bulk-flow velocity of the superfluid, and $\dot{s}_1$ and $\dot{s}_2$ represent the velocities with which local elements of vortex line will move. The ends of the vortex lines spend most of the time pinned to small protuberances on the walls of the flow channel as shown but, for sufficiently large values of v_a, they can escape, sliding along the walls until they get caught again by new pinning centres.

direction are dissipative processes. Furthermore, the effect of the flow on a loop of vortex pinned at its ends on small wall protuberances will be to make it expand and move as shown in *a* in the figure. Eventually, for a sufficiently fast flow velocity, the distorted loop 'de-pins' so that its ends can slide over the walls to 're-pin' on new sites, and the process repeats. The loop can thus travel dissipatively right across the channel. When it finally gets driven into the corners on the other side then, according to Schwarz's computer simulations, for sufficiently large flow velocities it re-orientates and re-connects (*b*) to form one or more small pinned loops that are suitably orientated for the initiation of new dissipative processes.

Schwarz suggests that the Avenel–Varoquaux observations can be accounted for if, for flow velocities just above those needed for de-pinning, vortices move across the orifice and then become pinned in the opposite corners in each half-cycle of the oscillating flow, so that they move across and back again once in each full cycle. It is an ingenious idea, but further work is needed to establish whether it describes what actually happens. Although the mechanism provides a plausible explanation of many features of the experimental results, there are other features that are less easily encompassed by the model, the most notable of which is the observation by Varoquaux, Meisel and Avenel that the experimental value of the critical velocity increases linearly with decreasing temperature. There seems to be no obvious reason why such behaviour should be expected on the basis of Schwarz's model. The experimental situation is not clear-cut, as the reproducibility of the data still has to be tested using different orifices, and the possibility of a microscopic leak in parallel with the orifice yet to be investigated. Further developments are awaited with interest. □

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Microscopy

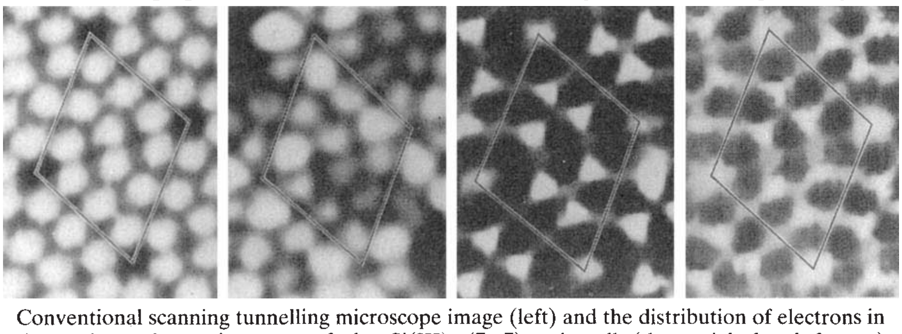
New ways to see atoms

John Pethica

The scanning tunnelling microscope developed by Binnig and Rohrer, who won the 1986 Nobel prize for Physics for this work (see *Nature* **323**, 663; 1986), is now well known for its ability to map the positions of atoms on surfaces[1]. On semiconductor surfaces, simultaneous spectroscopy of the tunnelling current allows the real-space location of particular electronic states or bonds to be shown in a dramatic manner[2] (see figure). Two important extensions of scanning tunnelling microscopy have recently been established. First, the direct writing of nearly single-atom structure on semiconductors, which is described by J. Golovchenko and colleagues on page 419 of this issue[3]; and second, the atomic force microscope and imaging of the atomic structure of insulators.

The size of semiconductor devices is constantly being reduced in search of greater density of packing and of new physical phenomena to use in their operation. The limiting scale imposed by atomic structure now seems to have been reached for a memory device by Golovchenko and co-workers[3], who show that a scanning tunnelling microscope can be used to 'write' a feature 8 Å in diameter and about 1 Å high onto an otherwise atomically smooth germanium surface. The presence of the feature can be subsequently 'read' using the microscope in its normal imaging mode. The mechanism of writing involves raising the tip voltage of the microscope to about 4 V at the desired location. A distinct shift in apparent width of the tunnel gap indicates the point at which material is transferred from the tip to the germanium surface; the width of the gap remains stable on the surface as the voltage is then re-lowered. Structure within the written, disk-like feature has not yet been resolved, but its size and shape suggest that at most one or two atoms have been added to the surface and the immediate sub-surface region of the germanium. Curiously, it was not possible to make such a surface modification on silicon; evidently the stability and energetics of the surface atomic structure are critical. The mechanism of writing is not yet understood, but probably involves some form of field evaporation. It seems to be necessary to 'recharge' the tip by bringing it into contact with (a distant region of) the germanium surface before the writing process. Although many aspects of the process are not understood, it is likely that the smallest possible randomly addressable 'bit' of information has now been written.

The ability to vary the tunnel gap width in scanning tunnelling microscopy allows the determination of the electron decay length into vacuum, and hence of the tunnel barrier height. An early problem in the technique was that improbably low



Conventional scanning tunnelling microscope image (left) and the distribution of electrons in the various electronic states of the Si(III)–(7×7) unit cell (three right-hand frames) measured simultaneously. This technique allows the electronic structure of the surface to be mapped out in real space; in other words, the chemical bonds may be seen. From ref. 2, courtesy of Robert Hamers.